Auctions with Selective Entry*

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Abstract

Entry has long been recognized to be an important part of many auction processes. In this paper, we consider auctions with entry based on a general analytical framework we call the Arbitrarily Selective (AS) model. This framework places minimal restrictions on the relationship between bidders' pre-entry information and their post-entry values, thereby nesting several important prior contributions (Levin and Smith (1994), Samuelson (1985)) as special cases. We characterize symmetric equilibrium in a broad class of standard auctions within this framework, in the process extending the classic revenue equivalence results of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to environments with endogenous and arbitrarily selective entry. We also explore the relationship between revenue maximization and efficiency in this environment, and show that Levin and Smith (1994)'s finding that revenue maximization implies efficiency applies only in the knife edge case of nonselective entry of Levin and Smith (1994).

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1 Introduction

Entry is a quantitatively and qualitatively important aspect of many real-world auction processes, but theoretical analysis of auctions with entry has primarily been limited to a few notable but restrictive special cases. Two paradigmatic examples in the literature are Samuelson (1985) (henceforth S), who proposes a simultaneous entry model in which potential bidders know their valuations *ex ante* but must incur a fixed cost to submit bids, and Levin and Smith (1994) (henceforth LS), who consider simultaneous entry under the alternative assumption that bidders learn their valuations after incurring the fixed cost. A common theme in this literature is that different assumptions on entry can produce very different practical and policy conclusions. For example, under the LS model a revenue-maximizing seller will set a zero reserve price and maximize social welfare, whereas in the S model revenue maximization requires a binding, socially inefficient reserve price. Hence while the existing literature contains many important insights on auctions with entry, it permits few overarching theoretical and policy conclusions.

This paper seeks to generalize several core results on auctions with entry to a framework we call the *Arbitrarily Selective (AS)* model. First suggested by Ye (2007) and subsequently explored by Marmer et al. (2013), Roberts and Sweeting (2013), Gentry and Li (2014), Bhattacharya and Sweeting (2015), and Lu and Ye (2015) among others, the AS model assumes that potential bidders receive imperfect signals of their valuations prior to entry, make simultaneous entry decisions based on these signals, then learn their valuations and submit bids. This structure imposes minimal *a priori* restrictions on pre-entry information, requiring only that higher signals lead bidders to expect stochastically higher post-entry valuations. It also nests both the S and LS models as special cases: the former when signals and values are perfectly correlated, and the latter when signals and values are independent. The AS model thus represents an ideal basis for general theoretical statements regarding properties of auctions with entry.

Motivated by these considerations, this paper makes the following specific contributions.

We extend the standard independent private values auction environment to accommodate endogenous and selective (AS) entry, and consider optimal design within a class of mechanisms we call *standard auctions with simultaneous entry* in the sense of Levin and Smith (1994).[1] For this class of auctions, we establish the following three results. First, we formally extend the classic revenue equivalence theorem of Myerson (1981), Riley and Samuelson (1981), and Levin and Smith (1994) to environments with endogenous and selective (AS) entry. Second, we characterize the efficient mechanism within the class of standard auctions with free entry, and show that the seller's revenue-maximizing auction will be inefficient in general. The latter result contrasts sharply with Levin and Smith (1994)'s finding that revenue maximization implies efficiency, further clarifying the sense in which this claim depends pivotally on the "knife edge" informational assumption of LS entry model. Nevertheless, we would like to emphasize that Levin and Smith (1994) have long recognized that the congruence between revenue and efficiency would fail when asymmetry among bidders or affiliated values are introduced. Finally, we explore revenue-maximizing reservation prices and entry fees directly, and establish that these will be positive in general. To our knowledge none of these results have been established at the level of generality we consider.

This study builds on and extends a substantial literature on auctions with endogenous entry. In addition to the studies cited above, notable early theoretical contributions to this literature include McAfee and McMillan (1987) and Smith and Levin (1996); the former explore a model of sequential entry where entry is interpreted as value discovery, the latter show that entry can lead a second-price auction to revenue dominate a first-price auction even when bidders are risk averse. In more recent work, Lu (2010) and Moreno and Wooders (2011) explore an extended version of the basic LS model in which bidders have private entry costs. Lu characterizes equilibrium, efficiency, and optimal auction design in this

[1] Roughly, this class of auctions consists of mechanisms such that only the highest bidder has a positive probability of award, and the probability of award depends only on the highest bid. We borrow the label *standard auctions with simultaneous free entry* from Bhattacharya and Sweeting (2015), who compare auctions with free entry with a range of other mechanisms by which the seller might attempt to (explicitly or implicitly) regulate entry. In Bhattacharya and Sweeting (2015), "free entry" means that potential bidders simultaneously and non-cooperatively decide whether to enter.

extended model, while Moreno and Wooders note that in the presence of private entry costs a revenue-maximizing seller will no longer achieve efficiency if ex ante entry fees are not allowed. In conjunction with our analysis, the latter observation suggests that the link between revenue maximization and efficiency highlighted by Levin and Smith (1994) is properly viewed as a "knife edge" case in the sense that any departure (in the direction of either private costs or private signals) will lead to failure of the result. Xu et al. (2013) study auctions with resale in a setting where bidders have either high or low entry costs and know their valuations before entry, showing that resale may introduce speculative motivations for entry, with ambiguous effects on efficiency and welfare. Finally, Bhattacharya and Sweeting (2015) explore the broader mechanism design implications of endogenous and selective (AS) entry. Bhattacharya and Sweeting (2015) show that the seller can often improve both revenue and efficiency by switching to one of several mechanisms which regulate entry in ways not permitted by the class of standard auctions with free entry. The current study complements this observation by providing a set of analytical results on optimal revenue and efficiency within the class of standard auctions with free AS entry, clarifying in particular how these differ from those obtained in the S and LS special cases.

Although our analysis here is primarily theoretical, our investigation is motivated by a substantial empirical literature on auctions with entry. Early work in literature establishes the relevance of entry in a wide range of applications: Bajari and Hortacsu (2003) in online auctions, Hendricks et al. (2003) in outer continental shelf "wildcat" auctions, Li and Zheng (2009) and Krasnokutskaya and Seim (2011) in highway construction procurement auctions, and Li and Zheng (2012), Li and Zhang (2015), Athey et al. (2011) and others in timber auctions, to mention just a few. More recently, a smaller literature has developed exploring empirical properties of the AS model specifically: notable contributions to this literature include Marmer et al. (2013), Gentry and Li (2014), Roberts and Sweeting (2013), and Bhattacharya et al. (2014) explore specification testing, nonparametric identification, and empirical applications of the AS model respectively. This study provides a theoretical counterpart to this recent application-oriented work.

Finally, although less immediately related, this study also contributes to a vast literature on mechanism design more generally. While we do not attempt to survey this literature in detail, seminal contributions include Vickery (1961), Clarke (1971), Groves (1973), Myerson (1981), Milgrom and Weber (1982), Ledyard (1978, 1980), and Ledyard and Groves (1980, 1988) among many others. Relative to the branch of this literature (e.g. Myerson (1981)) which focuses on auctions with independent private values, the main design challenge induced by endogenous and selective (AS) entry is that both the number and the composition of participants will change systematically with respect to the potential auction format employed. Hence existing optimality results (e.g. Myerson (1981)) do not directly apply and an alternative characterization accounting for both entry and selection is required.

The rest of the paper is organized as follows. Section 2 outlines the structure of the AS model, and Section 3 characterizes symmetric equilibrium entry as well as bidders' equilibrium payoffs and information rent under standard auction rules. In this section, we will also establish revenue equivalence in the class of auctions considered. Section 4 establishes that the seller's optimal auction will in general be inefficient, and Section 5 explores revenue-maximizing policies explicitly. Finally, Section 6 concludes.

2 The Arbitrarily Selective (AS) model

We study an auction model of a single indivisible good with endogenous entry. There are one seller and N potential bidders who have independent private values for the good being sold. The seller and all potential bidders are risk-neutral. Timing of the auction game is as follows. First, in Stage 1, each potential bidder i observes a private signal s_i of her (yet unknown) private value v_i, which falls in $\mathcal{V} = [0, \overline{v}]$, and all potential bidders simultaneously choose whether to enter the auction. Each entering bidder must pay an entry cost $c(> 0)$, which may be interpreted as the net of opportunity, learning, and bid preparation costs.

The seller may impose an entry fee/subsidy e to an entrant.[2] In Stage 2, the n bidders who chose to enter in Stage 1 learn their true values v_i and submit bids for the object being sold. Auction outcomes (allocation and payments) are determined according to a standard auction mechanism M, which will be formulated in Definition 1 and is common knowledge to all potential bidders. Seller's value is $v_0 \in \mathcal{V} = [0, \overline{v}]$. The entry fee/subsidy e and the standard auction mechanism M are instruments to be chosen optimally by the seller to maximize his expected revenue.

The value-signal structure and information structure of the *Arbitrarily Selective (AS)* entry model are further detailed in the following assumptions.

Assumption 1. *Each bidder i draws value-signal pairs (V_i, S_i) from a joint cumulative distribution $F(v, s)$ with density $f(v, s)$ satisfying the following properties:*

(i) The support of the random variable V_i is a bounded interval $\mathcal{V} = [0, \overline{v}]$, and the joint density distribution $f(v, s)$ is continuous.

(ii) For each bidder i, the conditional distribution of V_i is stochastically ordered in S_i: $s' \geq s$ implies $F(v|s') \leq F(v|s)$.

(iii) The random pairs (V_i, S_i) are independent across bidders: $(V_i, S_i) \perp (V_j, S_j)$ for all $j \neq i$.

(iv) Without loss of generality, we normalize first-stage signals S_i to have a uniform marginal distribution on $[0, 1]$: $S_i \sim U[0, 1]$.

The stochastic ordering condition in Assumption 1(ii) ensures that higher signals are "good news" in the sense of leading bidders to expect (weakly) stochastically higher distributions of valuations, but otherwise imposes minimal restrictions on the nature of selection. In particular, it nests both the S model (perfect dependence) and the LS model (independence) as special cases. The LS model is nested as a "knife edge" case by setting $V_i \perp S_i$; the S model is a limiting case where S_i fully determines V_i.

[2]A positive (resp. negative) e is interpreted as an entry fee (resp. subsidy).

Assumption 2. *Information structure:*

(i) Each bidder i observes own signal s_i prior to entry, but does not learn own value v_i until after entry.

(ii) The number of potential bidders N is known to all participants; the number of entrants n is either hidden until the auction concludes or revealed to all entrants before their bidding decisions are made.

Assumption 2(*ii*) ensures that the entrants' information on entry is symmetric among themselves, which entails a symmetric monotonic bidding strategy among entrants.

In the spirit of Riley and Samuelson (1981) and Levin and Smith (1994), we frame our analysis in terms of a general class of mechanisms we call *standard auctions*:

Definition 1. A *standard auction M* is any auction mechanism having the following properties:

1. Mechanism rules are anonymous.

2. If award of the good to an entrant is made, it is to the entrant submitting the highest bid.

3. The probability of award depends only on the highest bid, the award probability weakly increases with the highest bid.

4. For any symmetric distribution of values among entrants and any distribution of the number of entrants, there exists a unique symmetric strictly increasing bidding equilibrium.

5. An entrant with the lowest value gets non-negative finite expected payoff $\pi_M(0, n)$ for each number of entrants (i.e. n); $\pi_M(0, n)$ weakly decreases with n, and it is independent of the value distribution of entrants and the distribution of number of entrants.

The standard auctions defined above cover most mechanisms commonly used in practice. In particular, they cover first-price, second-price and all-pay auctions with a public or a random secret reservation price.

As we accommodate a general class of standard auctions as defined above, we frame our analysis in terms of direct mechanisms. By the Revelation Principle, any mechanism has an equivalent truthful direct mechanism, therefore there is no loss of generality. By Assumption 2(*ii*) (information structure) and Definition 1 (standard auctions), the good can be awarded only to the entrant with the highest value. Let the *award rule* $\alpha_M(y)$ denote the probability that mechanism M results in a sale when the highest (truthfully reported) value among entrants is y. From Definition 1 (part 3), the award rule $\alpha_M(y)$ is weakly increasing in the maximum entrant value y.

3 Equilibrium

On the equilibrium path, a symmetric equilibrium of a standard auction involves two components : a Stage 1 *entry threshold* s^* such that a potential bidder i enters if and only if $s_i \geq s^*$; and an anonymous Stage 2 equilibrium *bidding function*, which is monotonic in value and possibly depends on the number of entrants (i.e. n) if and only if n is disclosed publicly before their bidding decisions are made. Based on Definition 1, such an equilibrium bidding function must exist.

Note that a different standard auction (e.g. a second price auction versus a first price auction) would lead to different Stage 2 bidding strategy for any given entry threshold $\bar{s}$. In this sense, without further specifying the payment rule of the standard auction, we cannot pin down the bidding equilibrium. On the other hand, there is no need to pin down the bidding equilibrium in our analysis of efficient and revenue-maximizing auctions, as will be revealed in the following analysis.

For a standard auction M and an entry threshold $\bar{s}$, following the well-known usual revenue equivalence result of Myerson (1981), we can pin down the expected payoff of the threshold type $\bar{s}$ by using only the equilibrium allocation rule $\alpha_M(\cdot)$ and equilibrium payoffs $\pi_M(0, n)$ of the lowest value type. Setting this expected payoff of the threshold type $\bar{s}$ to $c + e$ (i.e. sum of entry cost and entry fee) would allow us to pin down the equilibrium entry threshold s^*.

3.1 Stage 2: Entrant payoff for given entry threshold $\bar{s}$

Suppose that in Stage 1 each potential bidder chooses to enter if and only if $s_i \geq \bar{s}$. Then the (selected) cumulative value distribution function of a representative entrant is given by

$$F^*(v; \bar{s}) \equiv \frac{1}{1 - \bar{s}} \int_{\bar{s}}^{1} F(v|s)\, ds, \tag{1}$$

where $F(v|s)$ stands for the a potential bidder's cumulative value distribution function conditional on signal s. $F^*(v; \bar{s})$ is stochastically increasing in $\bar{s}$ by Assumption 1(ii).

By Definition 1, an entrant i with value v will win against potential bidder j in one of two events: either j does not enter, or bidder j enters but draws a value less than v. Let $F^*_w(v; \bar{s})$ denote the joint probability of these events:

$$F^*_w(v; \bar{s}) = \bar{s} + (1 - \bar{s}) \cdot F^*(v; \bar{s}).$$

Differentiating $F^*_w(v; s)$ with respect to $\bar{s}$ we obtain:

$$\frac{\partial}{\partial \bar{s}} F^*_w(v; \bar{s}) = 1 - F(v|\bar{s}) \geq 0.$$

Hence the distribution $F^*_w(v; \bar{s})$ is stochastically decreasing in $\bar{s}$, a fact we will reference repeatedly in the derivations below.

The form of the equilibrium bidding function will obviously depend on the payment rule of the mechanism M, which is not specified in Definition 1 as it covers a wide spectrum of standard auctions. Nevertheless, via standard arguments in mechanism design, we can characterize an entrant's expected Stage 2 *payoff* in any standard auction as follows.

Proposition 1. *For a given entry threshold $\bar{s}$, in any symmetric monotone Stage 2 bidding equilibrium of any standard auction mechanism M, the expected Stage 2 payoff of an entrant with value v is given by*

$$\pi_M(v;\bar{s},N) = \int_0^v \alpha_M(y)\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N), \tag{2}$$

where $\pi_M(0;\bar{s},N) = \sum_{n=0}^{N-1} p(n;\bar{s},N-1)\pi_M(0,n+1)$ is an entrant's expected payoff if her value is 0, in which $p(n;\bar{s},N-1) = C_{N-1}^{n}(1-\bar{s})^n\bar{s}^{(N-1)-n}$ is the probability that an entrant faces n rivals in Stage 2 bidding competition.

Proposition 1 immediately follows Lemma 1 of Myerson (1981), which says that the derivative of expected payoff of a bidder with respect to their own value is simply the expected winning probability. In our environment, an entrant with value v wins with probability $\alpha_M(y)\cdot F_w^*(y;\bar{s})^{N-1}$ for given entry threshold $\bar{s}$.

3.2 Stage 1: Equilibrium entry threshold s^*

Given the Stage 2 payoff $\pi_M(v;\bar{s},N)$, we next characterize the symmetric Stage 1 equilibrium entry threshold s^*. Toward this end, consider the Stage 1 decision faced by potential bidder i with signal s_i facing $N-1$ potential rivals who enter according to $\bar{s}$. Bidder i's *ex ante* expected Stage 2 payoff if she enters is given by

$$\begin{aligned}
\Pi_M(s_i;\bar{s},N) &= E_v[\pi_M(v;\bar{s},N)|s_i] \\
&= \int_0^{\bar{v}} f(v|s_i)\int_0^v \alpha_M(y)\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N) \\
&= \int_0^{\bar{v}} \alpha_M(y)\cdot[1-F(y|s_i)]\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N),
\end{aligned}$$

where the second line follows from Proposition 1 and the third follows from integration by parts. The key properties of this ex ante profit function are stated in the following lemma.

Lemma 1. *Given entry threshold $\bar{s}$ and standard auction M,* ex ante *expected Stage 2 profit for an entrant with Stage 1 signal s_i is*

$$\Pi_M(s_i;\bar{s},N) = \int_0^{\bar{v}} \alpha(y)\cdot[1-F(y|s_i)]\cdot F_w^*(y;\bar{s})^{N-1}dy + \pi_M(0;\bar{s},N). \tag{3}$$

This function is weakly increasing in s_i for all $(\bar{s},N)$, strictly increasing in $\bar{s}$ for all (s_i,N), and strictly decreasing in N for all s_i and any $\bar{s}<1$.

Proof. The monotonicity of $\Pi_M(s_i;\bar{s},N)$ with respect to s_i is straightforward. Recall that function $F_w^*(y;\bar{s})^{N-1}$ increases with $\bar{s}$ and it is clear that it decreases with N. We next show that $\pi_M(0;\bar{s},N)$ also increases with $\bar{s}$ and decreases with N. Recall

$$\pi_M(0;\bar{s},N) = \sum_{n=0}^{N-1} C_{N-1}^n (1-\bar{s})^n \bar{s}^{(N-1)-n} \pi_M(0,n+1).$$

We have

$$\begin{aligned}
\frac{\partial \pi_M(0;\bar{s},N)}{\partial \bar{s}} &= \sum_{n=0}^{N-2} \frac{(N-1)!}{[(N-1)-n]!n!}\{(1-\bar{s})^n[(N-1)-n]\bar{s}^{(N-1)-n-1}\pi_M(0,n+1) \\
&\quad - \sum_{n=1}^{N-1} \frac{(N-1)!}{[(N-1)-n]!n!} n(1-\bar{s})^{n-1}\bar{s}^{(N-1)-n}\pi_M(0,n+1) \\
&= \sum_{n=0}^{N-2} \frac{(N-1)!}{[(N-1)-n]!n!}\{(1-\bar{s})^n[(N-1)-n]\bar{s}^{(N-1)-n-1}\pi_M(0,n+1) \\
&\quad - \sum_{n=1}^{N-1} \frac{(N-1)!}{[(N-1)-n]!n!} n(1-\bar{s})^{n-1}\bar{s}^{(N-1)-n}\pi_M(0,n+1) \\
&= \sum_{n=1}^{N-1} \frac{(N-1)!}{[(N-1)-n]!(n-1)!}(1-\bar{s})^{n-1}\bar{s}^{(N-1)-n}\pi_M(0,n) \\
&\quad - \sum_{n=1}^{N-1} \frac{(N-1)!}{[(N-1)-n]!(n-1)!}(1-\bar{s})^{n-1}\bar{s}^{(N-1)-n}\pi_M(0,n+1) \\
&= \sum_{n=0}^{N-1} \frac{(N-1)!}{[(N-1)-n]!(n-1)!}(1-\bar{s})^{n-1}\bar{s}^{(N-1)-n}[\pi_M(0,n)-\pi_M(0,n+1)] \\
&\geq 0.
\end{aligned}$$

Therefore, $\pi_M(0;\bar{s},N)$ also increases with $\bar{s}$. We now turn to the monotonicity of $\pi_M(0;\bar{s},N)$ with respect to N.

$$\begin{aligned}
& \pi_M(\underline{v};\bar{s},N+1) - \pi_M(\underline{v};\bar{s},N) \\
= & \sum_{n=0}^{N} C_N^n(1-\bar{s})^n\bar{s}^{N-n}\pi_M(\underline{v},n+1) - \sum_{n=0}^{N-1} C_{N-1}^n(1-\bar{s})^n\bar{s}^{(N-1)-n}\pi_M(\underline{v},n+1) \\
= & C_N^N(1-\bar{s})^N\pi_M(\underline{v},N+1) + \sum_{n=0}^{N-1}[C_N^n(1-\bar{s})^n\bar{s}^{N-n} - C_{N-1}^n(1-\bar{s})^n\bar{s}^{(N-1)-n}]\pi_M(\underline{v},n+1) \\
= & C_N^N(1-\bar{s})^N\pi_M(\underline{v},N+1) + \sum_{n=0}^{N-1}(\frac{N}{N-n}\bar{s}-1)C_{N-1}^n(1-\bar{s})^n\bar{s}^{(N-1)-n}\pi_M(\underline{v},n+1).
\end{aligned}$$

Let $n^* = \min\{n | \frac{N}{N-n}\bar{s} - 1 \leq 0, 0 \leq n \leq N-1\}$. Thus

$$
\begin{aligned}
& \pi_M(0;\bar{s}, N+1) - \pi_M(0;\bar{s}, N) \\
\leq \;& C_N^N(1-\bar{s})^N \pi_M(0, n^*+1) + \sum_{n=0}^{n^*}(\frac{N}{N-n}\bar{s} - 1)C_{N-1}^n(1-\bar{s})^n \bar{s}^{(N-1)-n}\pi_M(0, n^*+1). \\
& + \sum_{n=n^*}^{N-1}(\frac{N}{N-n}\bar{s} - 1)C_{N-1}^n(1-\bar{s})^n \bar{s}^{(N-1)-n}\pi_M(0, n^*+1) \\
= \;& \pi_M(0, n^*+1)[\sum_{n=0}^{N} C_N^n (1-\bar{s})^n \bar{s}^{N-n} - \sum_{n=0}^{N-1} C_{N-1}^n (1-\bar{s})^n \bar{s}^{(N-1)-n}] \\
= \;& 0.
\end{aligned}
$$

∎

Bidder i will choose to enter whenever expected net profit from entry is positive, i.e. whenever

$$\Pi_M(s_i;\bar{s}, N) \geq c + e. \tag{4}$$

This fact in turn implies a break-even condition which must hold at any candidate *interior* equilibrium $s^* \in (0,1)$:

$$\Pi_M(s^*; s^*, N) \equiv c + e,$$

that is, a bidder drawing signal $S_i = s^*$ must be indifferent to entry when facing $N-1$ potential rivals who also enter according to s^*. Noting that $\Pi_M(s_i;\bar{s}, N)$ is increasing in $(s_i, \bar{s})$, we ultimately conclude:[3]

Proposition 2. *A symmetric entry equilibrium in the AS model is characterized by a signal threshold s^* such that only bidders with $s_i \geq s^*$ choose to enter. This signal threshold is uniquely determined as follows.*

- *If $\Pi_M(0;0,N) > c + e$, then $s^* = 0$ and all potential bidders always enter.*

[3] A formal proof is omitted to save space. Please refer to the proof of Proposition 2 of Gentry and Li (2014) for details. In particular, Assumption 1(i) guarantees continuity of $\Pi_M(\bar{s};\bar{s}, N)$ in $\bar{s}$.

- *If* $\Pi_M(1;1,N) < c+e$, *then* $s^* = 1$ *and no potential bidder ever enters.*

- *Otherwise, the signal threshold* s^* *satisfies the break-even condition*

$$\Pi_M(s^*;s^*,N) \equiv c+e, \tag{5}$$

where $\Pi_M(\cdot;\cdot,\cdot)$ *is defined as in Lemma 1.*

Furthermore, considered as a function of $(N, c+e)$, *the equilibrium threshold* $s_N^*(c+e)$ *satisfies the following monotonicity properties:*

- *For any* $N \geq 1$, $s_N^*(c+e)$ *is continuous and weakly increasing in* $c+e$, *with strict monotonicity whenever* $s_N^*(c+e) \in (0,1)$.

- *For any* c, *we have* $N' > N$ *implies* $s_{N'}^*(c+e) \geq s_N^*(c+e)$. *If in addition* $s_N^*(c+e) \in (0,1)$, *then* $s_{N'}^*(c+e) > s_N^*(c+e)$ *and* $s_{N'}^*(c+e) \in (0,1)$.

Proposition 2 characterizes the unique symmetric entry equilibrium of the AS model under any standard auction with simultaneous entry.

3.3 Information rent

Lemma 1 and Proposition 2 allow us to pin down the information rent of a first stage type $s_i (> s^*)$:

$$\begin{aligned} \Delta\pi_M(s_i, s^*) &= \Pi_M(s_i;s^*,N) - \Pi_M(s^*;s^*,N) \\ &= \int_0^{\bar{v}} \alpha_M(y) \cdot [F(y|s^*) - F(y|s_i)] \cdot F_w^*(y;s^*)^{N-1} dy \\ &\geq 0. \end{aligned}$$

Therefore, we have

$$\frac{\partial \pi_M(s_i, s^*)}{\partial s_i} = -\int_0^{\bar{v}} \alpha_M(y) \cdot F_{s_i}(y|s_i) F_w^*(y;s^*)^{N-1} dy \geq 0,$$

which says that the information rent is at least weakly increasing with the first stage type s_i, with strict inequality if and only if $F_{s_i}(y|s_i) < 0$ for some $y \in [0, \bar{v}]$.

Lemma 2. *The ex ante expected surplus of a potential bidder is*

$$\begin{aligned}\Pi_M(s^*) &= \int_{s^*}^{1} \Delta\pi_M(s_i, s^*) ds_i \\ &= (1-s^*)\int_0^{\bar{v}} \alpha_M(y)\cdot[F(y|s^*) - F^*(y;s^*)]F_w^*(y;s^*)^{N-1}dy. \qquad (6)\end{aligned}$$

Proof. Note that

$$\begin{aligned}\int_{s^*}^{1} \Pi_M(s_i; s^*, N) ds_i &= \int_{s^*}^{1}\int_0^{\bar{v}} \alpha_M(y)\cdot[1 - F(y|s_i)]\cdot F_w^*(y;s^*)^{N-1}dyds_i \\ &= \int_0^{\bar{v}} \alpha_M(y)\cdot\{\int_{s^*}^{1}[1-F(y|s_i)]ds_i\}\cdot F_w^*(y;s^*)^{N-1}dy \\ &= \int_0^{\bar{v}} \alpha_M(y)\cdot[(1-s^*) - \int_{s^*}^{1} F(y|s_i)ds_i]\cdot F_w^*(y;s^*)^{N-1}dy \\ &= \int_0^{\bar{v}} \alpha_M(y)\cdot(1-s^*)[1 - \frac{\int_{s^*}^{1} F(y|s_i)ds_i}{1-s^*}]\cdot F_w^*(y;s^*)^{N-1}dy \\ &= \int_0^{\bar{v}} \alpha_M(y)\cdot(1-s^*)[1-F^*(y;s^*)]\cdot F_w^*(y;s^*)^{N-1}dy,\end{aligned}$$

and

$$\begin{aligned}\int_{s^*}^{1} \Pi_M(s^*; s^*, N) ds_i &= \int_{s^*}^{1}\int_0^{\bar{v}} \alpha_M(y)\cdot[1 - F(y|s^*)]\cdot F_w^*(y;s^*)^{N-1}dyds_i \\ &= \int_0^{\bar{v}} \alpha_M(y)\cdot\{\int_{s^*}^{1}[1-F(y|s^*)]ds_i\}\cdot F_w^*(y;s^*)^{N-1}dy \\ &= \int_0^{\bar{v}} \alpha_M(y)\cdot[(1-s^*) - \int_{s^*}^{1} F(y|s^*)ds_i]\cdot F_w^*(y;s^*)^{N-1}dy \\ &= \int_0^{\bar{v}} \alpha(y)\cdot(1-s^*)[1-F^*(y|s^*)]\cdot F_w^*(y;s^*)^{N-1}dy.\end{aligned}$$

∎

Note that according to (6), bidders' information rent depends on e and $\pi_M(0,n)$ only through their impacts on the entry threshold s^*. It is clear that $\Pi_M(s^*)$ would be zero if and only if our model reduces to the "knife edge" case of LS where $F(y|s^*) - F^*(y; s^*) = 0$. In Lemma 4, we will further study how the information rent changes with seller instruments including entry fee and reservation price.

3.4 Revenue equivalence

We are now ready to formally extend the seminal revenue equivalence result of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to accommodate endogenous and selective (AS) entry. By definition, for a standard auction M with entry fee/subsidy e which induces equilibrium entry s^*, *ex ante* expected seller revenue is the difference between total social welfare and surplus accruing to bidders:

$$R_M(s^*) = TS_M(s^*) - N\Pi_M(s^*),$$

where $TS_M(s^*)$ denotes expected total surplus generated, and $\Pi_M(s^*)$ is the expected *ex ante* equilibrium payoff for any given potential bidder at equilibrium, which we have identified in Lemma 2. Note that like bidders' information rent, social welfare TS_M depends on e and $\pi_M(0, n)$ only through their impacts on the entry threshold s^*.

It is clear that under standard auction M and equilibrium entry s^*, we have

$$TS_M(s^*) = \int \{y\alpha_M(y) + v_0(1 - \alpha_M(y))\} d[F_w^*(y; s^*)^N] - N(1 - s^*)c.$$

By Proposition 2, any two mechanisms M_1 and M_2 having the same award rule and payoff of the lowest-value type must induce the same equilibrium entry. Revenue equivalence of M_1 and M_2 then follows immediately as both social welfare and bidders' rent are same across the two mechanisms. This result extends the classic equivalence results of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to the case of AS entry, and is formally stated in the following proposition.

Proposition 3 (Revenue Equivalence)**.** *Suppose standard auctions M_1 and M_2 implement the same award rule and render the same payoffs to the lowest-value type for each fixed n, and thus that they are revenue equivalent for each fixed n. Then for any entry fee/subsidy e, M_1 and M_2 are revenue-equivalent under AS entry.*

4 Efficiency versus revenue maximization

In this section we study the relationship between social efficiency and revenue maximization in the class of standard auctions with simultaneous AS entry. This investigation is motivated by a key result of Levin and Smith (1994): when bidders enter without selection, a revenue-maximizing seller will also maximize social welfare. We show that this conclusion applies only in the "knife edge" LS case: in the broader AS model, the revenue seller will generally prefer an inefficient mechanism. Based on the revenue equivalence result, we do not need to specify the payment rule of a standard auction in our analysis. For current purpose, we assume that the allocation of a standard auction is fully described by a public reserve price $r \in [0, \bar{v}]$. In other words, an entrant with the highest value wins if and only if her value is above r. Therefore, we focus on two policy instruments for the seller: a public reserve price r and an ex ante entry fee/subsidy e.

First consider social welfare. Recall that the entry fee/subsidy e and the lowest-valuation payoff function $\pi_M(0, n)$ affect total welfare only through their impacts on entry. Fix an arbitrary entry threshold s. Then for any r, the total welfare is

$$TS(s, r) = v_0 F_w^*(r; s)^N + \int_r^{\bar{v}} y d[F_w^*(y; s)^N] dy - N(1 - s)c. \tag{7}$$

Note that $\forall s$, we have $r = v_0$ maximizes $TS(s, r)$. We next identify entry s_e that maximizes $TS(s, v_0)$. Appropriate choices for e and $\pi_M(0, n)$ would induce s_e as an equilibrium entry threshold without affecting total welfare.

Rearranging (7) via integration by parts produces the following equivalent representation for social welfare at threshold s:

$$TS(s,v_0) = \bar{v} - \int_{v_0}^{\bar{v}} F_w^*(y;s)^N dy - N(1-s)c. \tag{8}$$

It can be shown that this function is concave in s, since $\frac{\partial TS(s,v_0)}{\partial s} = -N\int_{v_0}^{\bar{v}} F_w^*(y;s)^{N-1}[1 - F(y|s)]dy + Nc$, which decreases with s. Recall that $F_w^*(y;s)$ increases with s, and $F(y|s)$ decreases with s.

Social welfare is thus uniquely maximized at a threshold s_e satisfying the necessary and sufficient first-order condition[4]

$$-N\int_{v_0}^{\bar{v}} F_w^*(y;s_e)^{N-1}[1 - F(y|s_e)]dy + Nc \equiv 0. \tag{9}$$

As $\pi_M(0,n)$ does not affect $TS(s,r)$, we set $\pi_M(0,n) = 0, \forall n$. Then by Proposition 2, s_e must be the entry equilibrium when $e = 0$. This observation translates into the following characterization of the socially optimal mechanism:

Proposition 4 (Efficiency)**.** *Within the class of standard auctions with simultaneous entry, social welfare is maximized in any ex post efficient auction M (which renders $\pi_M(0,n) = 0, \forall n$) with zero ex ante entry fee.*

This result generalizes the findings of Levin and Smith (1994) and Lu (2010) on ex ante efficient auctions when players have to incur information costs to discover their values. However, unlike Levin and Smith (1994), this ex ante efficient auction is not revenue maximizing in general, as will be shown below.

Set $\pi_M(0,n) = 0$ and $r = v_0$. Now e is the seller's only policy choice. Let $s^*(e)$ denote the equilibrium entry threshold. We consider the seller's revenue maximization problem. By definition, seller revenue is the difference between social surplus and expected profits among

[4]Without loss of generality, we assume s_e is an interior solution.

potential bidders, which with slight abuse of notation we write as follows:

$$R(e) = TS(s^*(e), v_0) - N\Pi_M(s^*(e)).$$

The seller's optimal e^* satisfies

$$\frac{\partial R(e^*)}{\partial e} = 0,$$

i.e.,

$$\frac{\partial TS(s^*(e^*), v_0)}{\partial s}\frac{ds^*(e^*)}{de} - N\frac{d\Pi_M(s^*(e^*))}{de} = 0.$$

Note that when $e = 0$, we have $\frac{\partial TS(s^*(0),v_0)}{\partial s} = 0$ as $e = 0$ induces efficient entry as established above. We thus have

$$\frac{\partial R(0)}{\partial e} = -N\frac{d\Pi_M(s^*(0))}{ds}s^{*\prime}(0). \tag{10}$$

When entry is selective, neither $\frac{d\Pi_M(s^*(0))}{ds}$ nor $s^{*\prime}(0)$ in the RHS of (10) will be zero in general. Note that an interior s^* strictly decreases with total entry costs $c + e$. The monotonicity of Π_M with respect to s will be revealed by Lemma 4. Hence the seller's optimal policy need not correspond to the social optimum. We state this result formally as a lemma:

Lemma 3. *In general, a revenue-maximizing seller does not maximize social welfare.*

As noted above, this result contrasts with the corresponding finding in Levin and Smith (1994). Intuitively, when potential bidders have no private ex ante information that is correlated to their ex post values, bidder surplus will be identically zero for all (e, r), so social welfare and seller revenue coincide and a revenue-maximizing seller will maximize total surplus. In contrast, when entry is strictly selective, bidder surplus is positive and decreasing in the entry threshold s^* as will be revealed by Lemma 4. Therefore, a revenue-maximizing seller will need to induce distortion to capture part of this additional surplus.

5 Revenue-maximizing auctions

The last section proceeded via negation, first characterizing the socially efficient mechanism within the class of standard auctions with simultaneous free entry, then noting that in general a revenue-maximizing seller will not choose this auction. This section proceeds more positively, first characterizing revenue-maximizing choices of the seller's policy variables e and r. As in the last section, we let $\pi_M(0,n)=0$.

5.1 Unrestricted optimum

We first look at the scenario when the seller can choose both r and e. Since equilibrium entry s is uniquely determined by r and e, we can equivalently assume the seller uses r and s as instruments. Based on our entry equilibrium characterization in Proposition 2, we have that $e(s) \equiv \int_r^{\bar{v}} [1-F(y|s)] F_w^*(y;s)^{N-1} dy - c$ is the entry fee/subsidy required to induce threshold s as an entry equilibrium. In the following analysis, we will write social welfare, bidders' rent and seller revenue as functions of the instruments r and s.

Recall that by (7) social welfare is

$$\begin{aligned} TS(s,r) &= v_0 F_w^*(r;s)^N + \int_r^{\bar{v}} y d[F_w^*(y;s)^N] dy - N(1-s)c \\ &= \bar{v} + (v_0 - r) F_w^*(r;s)^N - \int_r^{\bar{v}} F_w^*(y;s)^N dy - N(1-s)c, \end{aligned}$$

that by Lemma 2 a potential bidder's information rent is

$$\Pi(s,r) = (1-s) \int_r^{\bar{v}} [F(y|s) - F^*(y;s)] F_w^*(y;s)^{N-1} dy,$$

and that seller revenue is given by

$$R(s,r) = TS(s,r) - N\Pi(s,r).$$

We first establish several properties of $TS(s,r)$ and $\Pi(s,r)$, which we state formally in the next lemma.

Lemma 4. *(i) Social welfare $TS(s,r)$ is maximized uniquely at $r = v_0$, $\forall s$; $\forall r \geq v_0$, $\frac{\partial TS(s,r)}{\partial s} \leq 0$.*

(ii) Bidder information rent $\Pi(s,r)$ decreases with r, s.

Proof. Note that

$$\frac{\partial TS(s,r)}{\partial r} = (v_0 - r)\frac{dF_w^*(y;s)^N}{dy}|_{y=r},$$

which means that $TS(s,r)$ is maximized uniquely at $r = v_0$, $\forall s$.

Note further that

$$\frac{\partial TS(s,r)}{\partial s} = (v_0 - r)NF_w^*(r;s)^{N-1}[1 - F(r|s)] - N\int_r^{\bar{v}} F_w^*(y;s)^{N-1}[1 - F(y|s)]dy + Nc.$$

Recall that $F_w^*(y;s)$ increases with s, while $F(y|s)$ decreases with s. Thus for $\forall r \geq v_0$, we have that $\frac{\partial TS(s,r)}{\partial s}$ decreases with s, i.e. that $TS(s,r)$ is concave in s.

Moreover, since $F(r|s) - F^*(r;s) \geq 0$, we have

$$\begin{aligned}\frac{\partial \Pi(s,r)}{\partial r} &= -(1-s)[F(r|s) - F^*(r;s)]F_w^*(r;s)^{N-1} \\ &\leq 0, \forall r \in [0, \bar{v}).\end{aligned}$$

In addition, we have

$$\frac{\partial \Pi(s,r)}{\partial s} = \int_r^{\bar{v}} \frac{d\{(1-s)[F(y|s) - F^*(y;s)]F_w^*(y;s)^{N-1}\}}{ds}dy \leq 0,$$

since

$$\begin{aligned}\frac{d\{F_w^*(y;s)^{N-1}\}}{ds} &= (N-1)F_w^*(y;s)^{N-2}\frac{dF_w^*(y;s)}{ds} \\ &= (N-1)F_w^*(y;s)^{N-2}(1-s)\frac{dF^*(y;s)}{ds} \\ &= -(N-1)F_w^*(y;s)^{N-2}F(y|s) \leq 0,\end{aligned}$$

and

$$\begin{aligned}
& \frac{d\{(1-s)[F(y|s)-F^*(y;s)]\}}{ds} \\
= & \frac{d\{(1-s)[F(y|s)-\frac{1}{1-s}\int_s^1 F^*(y;t)dt]\}}{ds} \\
= & \frac{d\{(1-s)F(y|s)-\int_s^1 F^*(y;t)dt]\}}{ds} \\
= & -F(y|s)+(1-s)\frac{dF(y|s)}{ds}+F(y|s) \\
= & (1-s)\frac{dF(y|s)}{ds} \leq 0.
\end{aligned}$$

∎

We now are ready to investigate revenue-maximizing choices for (s, r).

Proposition 5 (Optimal entry fee and reserve)**.** *The optimal entry fee e^* must be nonnegative and the optimal reserve r^* must weakly exceed v_0. Further, if entry is strictly selective, then both inequalities are strict.*

Proof. The results of Lemma 4 immediately mean that the revenue-maximizing reserve cannot be lower than the seller's value v_0. In particular, if $F(r|s) - F^*(r;s) > 0$, i.e. if a higher s is associated with a strictly stochastically dominant distribution $F(\cdot|s)$, then we have

$$\frac{\partial R(s,r)}{\partial r}|_{r=v_0} > 0, \forall s,$$

which means that the revenue-maximizing reserve r must be strictly above the seller's value v_0. Moreover, $\frac{\partial R(s,r)}{\partial r}|_{r=v_0} = 0, \forall s$ if and only if $F(\cdot|s)$ does not depend on s, which means we have an LS model.

At the optimum (s^*, r^*), we have the following first order conditions:

$$\frac{\partial R(s^*, r^*)}{\partial s} = \frac{\partial TS(s^*, r^*)}{\partial s} - \frac{\partial \Pi(s^*, r^*)}{\partial s} = 0;$$
$$\frac{\partial R(s^*, r^*)}{\partial r} = \frac{\partial TS(s^*, r^*)}{\partial r} - \frac{\partial \Pi(s^*, r^*)}{\partial r} = 0.$$

Recall that we have $r^* > v_0$ and $\frac{\partial \Pi(s,r)}{\partial r} < 0$, $\frac{\partial \Pi(s,r)}{\partial s} < 0$ unless we have the LS model, in which case $r^* = v_0$ and $\frac{\partial \Pi(s,r)}{\partial r} = 0$, $\frac{\partial \Pi(s,r)}{\partial s} = 0$. Note that

$$\frac{\partial TS(s, r)}{\partial s} = (v_0 - r)\frac{\partial F_w^*(r; s)^N}{\partial s} - N\{\int_r^{\bar{v}} F_w^*(y; s)^{N-1} dF_w^*(y; s) - c\}.$$

Therefore, at the optimum, we have

$$\frac{\partial TS(s^*, r^*)}{\partial s} = \frac{\partial \Pi(s^*, r^*)}{\partial s},$$

$$\text{i.e., } (v_0 - r^*)\frac{\partial F_w^*(r^*; s)^N}{\partial s} - N\{\int_r^{\bar{v}} F_w^*(y; s^*)^{N-1} dF_w^*(y; s^*) - c\} = \frac{\partial \Pi(s^*, r^*)}{\partial s},$$

$$\text{or } N\{\int_r^{\bar{v}} F_w^*(y; s^*)^{N-1} dF_w^*(y; s^*) - c\} = (v_0 - r^*)\frac{\partial F_w^*(r^*; s^*)^N}{\partial s} - \frac{\partial \Pi(s^*, r^*)}{\partial s} > 0.$$

Note that $(v_0 - r^*)\frac{\partial F_w^*(r^*;s^*)^N}{\partial s} > 0$ as $v_0 - r^* < 0$ and $\frac{\partial F_w^*(r^*;s^*)^N}{\partial s} < 0$. Recall further that the expected payoff of type s^* when the entry fee/subsidy e is set to zero is

$$\int_r^{\bar{v}} F_w^*(y; s^*)^{N-1} dF_w^*(y; s^*) - c.$$

For s^* to be an equilibrium, we must therefore have

$$e^* = \int_r^{\bar{v}} F_w^*(y; s^*)^{N-1} dF_w^*(y; s^*) - c = [(v_0 - r^*)\frac{\partial F_w^*(r^*; s^*)^N}{\partial s} - \frac{\partial \Delta\pi(s^*, r^*)}{\partial s}]/N > 0.$$

∎

5.2 Restricted optimums

We now turn to two restricted cases. In case I, we restrict the reserve price to be ex post efficient, i.e. $r = v_0$, and study the optimal entry fee/subsidy. In case II, we restrict the entry fee to be zero, and study the optimal reserve price.

5.2.1 Seller's optimal entry fee when $r = v_0$

Set $r = v_0$. We look at $\frac{\partial R(s,v_0)}{\partial s}$. By Proposition 4, $TS(s, v_0)$ is uniquely maximized at $s = s_e$, which is induced by $e = 0$. By Lemma 4, we have $\frac{\partial TS(s,v_0)}{\partial s} > 0$ for $s < s_e$, $\frac{\partial TS(s,v_0)}{\partial s} < 0$ for $s > s_e$, and $\frac{\partial \Pi(s,v_0)}{\partial s} \leq 0$, which jointly lead to $\frac{\partial R(s,v_0)}{\partial s} > 0$ for $s < s_e$. This means that the optimal s^* must be higher than s_e, which in turn implies that the optimal $e^* \geq 0$. Moreover, as $\frac{\partial \Pi(s,v_0)}{\partial s} < 0$ unless we have the LS model, which leads to $\frac{\partial R(s,v_0)}{\partial s} > 0$ for $s \leq s_e$. This means that unless we have the LS model, we must have that the optimal entry fee must be strictly positive when reserve is efficient. This result is formally stated in the following proposition.

Proposition 6 (Optimal entry fee)**.** *The optimal entry fee must be strictly positive when the reserve is efficient, unless we have the LS model.*

5.2.2 Seller's optimal reserve price when $e = 0$

Now set $e = 0$. We first show that the optimal reserve r cannot be lower than efficient level v_0. Note an r lower than v_0 leads to an equilibrium entry threshold $s(r)$ strictly lower than s_e, the efficient entry level. Note that $TS(s, r)$ is maximized at $(s_e, r = v_0)$. As a result, $TS(s(r), r)$, where $r < v_0$ is strictly lower than $TE(s_e, r = v_0)$.

Recall that Lemma 4 shows that $\Pi(s, r)$ decreases with both s and r. As a result, the optimal reserve cannot be lower than efficient level v_0; in this case, we would have both lower total surplus and higher total information rent (and hence lower seller revenue) due to the interaction between lower r and the associated lower s. Note further that $\frac{d\Pi(s(r),r)}{dr} = \frac{\partial \Pi(s(r),r)}{\partial r} + \frac{\partial \Pi(s(r),r)}{\partial s} s'(r)$, where $\frac{\partial \Pi(s(r),r)}{\partial r} \leq 0$, $\frac{\partial \Pi(s(r),r)}{\partial s} \leq 0$ and $s'(r) > 0$.

We next show the optimal r cannot be v_0 unless we have the LS model. For this purpose, we only need to show that $\frac{dTS(s(r),r)}{dr}|_{r=v_0} = 0$, since $\frac{d\Pi(s(r),r)}{dr} < 0$ as argued above unless we have the LS model. We have

$$\frac{dTS(s(r),r)}{dr} = \frac{\partial TS(s(r),r)}{\partial r} + \frac{\partial TS(s(r),r)}{\partial s} s'(r).$$

Recall

$$\frac{\partial TS(s(r),r)}{\partial r} = (v_0 - r)\frac{dF_w^*(y;s(r))^N}{dy}|_{y=r},$$

which leads to

$$\frac{\partial TS(s(r),r)}{\partial r}|_{r=v_0} = 0.$$

Moreover, it is clear that $\frac{\partial TS(s(r),r)}{\partial s}|_{r=v_0} = \frac{\partial TS(s_e,v_0)}{\partial s} = 0$ since $TS(s,v_0)$ is maximized at $s = s_e$. We thus indeed have $\frac{dTS(s(r),r)}{dr}|_{r=v_0} = 0$.

We thus have the following result.

Proposition 7 (Seller's optimal reserve price)**.** *The optimal reserve must be strictly above the efficient level (i.e. v_0) when there is no entry fee, unless we have the LS model.*

6 Conclusion

This study proposes a general analysis of auctions with entry based on a framework we call the Arbitrarily Selective (AS) model. From a theoretical perspective, this framework has several major advantages: it relaxes existing restrictions on pre-entry information, permits richer endogenous entry structures, and nests several leading special cases in the literature. The AS model thus represents an ideal basis for a general theoretical approach to auctions with entry.

Within this general AS entry framework, we focus on the broad class of standard auctions considered by Riley and Samuelson (1981) and Levin and Smith (1994): roughly, auctions such that award (if made) is only to the highest bidder. For this class of auctions, we establish the following three results. First, we extend the classic revenue equivalence results of Myerson (1981), Riley and Samuelson (1981) and Levin and Smith (1994) to auctions with endogenous and arbitrarily selective entry. Second, we characterize ex ante efficient auction in the AS model, and show that a revenue-maximizing seller will in general induce inefficient entry decisions. Finally, we explore revenue-maximizing policies in the AS model, and establish that the seller will prefer positive reservation prices and entry fees. While some of these results were available for special cases of our model, to our knowledge none have been established at the level of generality we consider.

Levin and Smith (1994) have long recognized that the congruence between revenue and efficiency would fail when asymmetry among bidders or affiliated values are introduced. Our findings on the sub-optimality of an ex ante efficient reservation price further reveal that if bidders hold pre-entry private information about their ex post values, coincidence between revenue-maximizing and efficient reservation prices in general would fail. This observation in turn has potentially important implications for both policy design and welfare analysis.

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